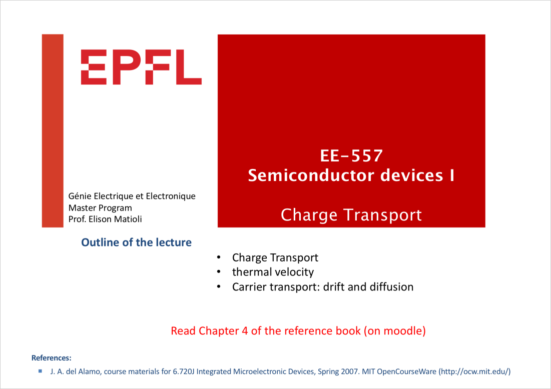
Course 8

Monday, 6 October 2025 14:53

EE557Cours 8 - Charge...



EE-557
Semiconductor devices I

Charge Transport

Génie Electrique et Electronique
Master Program
Prof. Elison Matioli

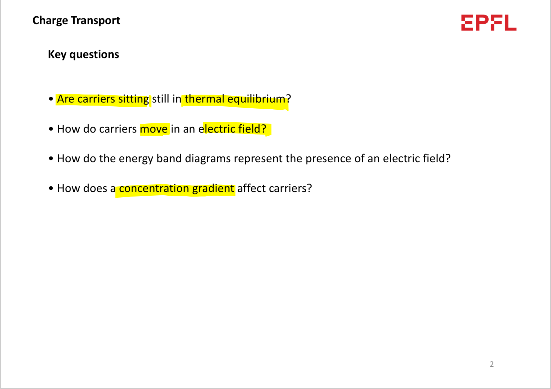
Outline of the lecture

- Charge Transport
- thermal velocity
- Carrier transport: drift and diffusion

Read Chapter 4 of the reference book (on moodle)

References:

- J. A. del Alamo, course materials for 6.720J Integrated Microelectronic Devices, Spring 2007. MIT OpenCourseWare (http://ocw.mit.edu/)



Charge Transport

Key questions

- Are carriers sitting still in thermal equilibrium?
- How do carriers move in an electric field?
- How do the energy band diagrams represent the presence of an electric field?
- How does a concentration gradient affect carriers?

Charge Transport

Carrier flow in semiconductors

Any **motion of free carriers** in a semiconductor leads to a **current**.

Drift:

This motion can be caused by an **electric field** due to an externally applied voltage (electric field), since the carriers are charged particles. We will refer to this transport mechanism as **carrier drift**.

Ex: n-channel MOSFET

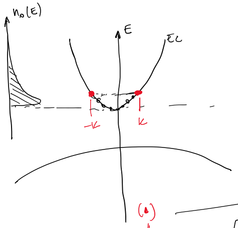
Diffusion:

In addition, carriers also move from regions where the **carrier density is high to regions where the carrier density is low**. This carrier transport mechanism is due to the thermal energy and the associated random motion of the carriers. We will refer to this transport mechanism as **carrier diffusion**.

Ex: npn bipolar transistors

The **total current** in a semiconductor equals the **sum** of the **drift and the diffusion** currents.

What happens with electrons at a given temperature without electric field?



Electron velocity (Group velocity):

$$v_n(k) = \frac{1}{\hbar} \nabla_k E_n(k)$$

At the bottom of conduction band:

$$(1) \quad E = \frac{\hbar^2 k^2}{2m^*} = \frac{p^2}{2m^*}$$

— momentum

$$v_n(k) = \frac{\hbar k}{m^*} = \frac{p}{m^*}$$

$$\underbrace{\vec{p} = \hbar \vec{k}}_{\text{quantum}} = \underbrace{m \cdot \vec{v}}_{\text{classic}}$$

$$v_n(k) = \frac{\hbar k}{m^*} \overset{(2)}{\Rightarrow} \boxed{v_n(E) = \sqrt{\frac{2(E-E_c)}{m^*}}}$$

Average thermal velocity:

moodle ⇓

$$\langle v_{th} \rangle = \frac{\int_{E_c}^{\infty} v_n(E) \cdot n_0(E)\, dE}{\int_{E_c}^{\infty} n_0(E)\, dE} = \sqrt{\frac{8\,kT}{\pi m^*_{ce}}}$$

Thermal motion and scattering

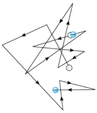
We can think of carriers as particles in **an ideal gas**.

At finite T, carriers have **finite thermal energy**. All this energy resides in the **kinetic energy** of the particles.

Carriers move in **random directions**: no net velocity, but average carrier velocity is thermal velocity:

$$v_{th} = \sqrt{\frac{8\,kT}{\pi\, m_c^*}}$$

Zero electric field



$m^*_{ce} \equiv$ **conductivity effective mass** [$eV \cdot s^2/cm^2$]

- This is the effective mass that connects the kinetic energy of an electrons with its velocity
- It accounts for all interactions between the carriers and the perfect periodic potential of the lattice.
- m^*_{ce} and m^*_{ch} are different from the effective mass from the density of states (you can find the values in appendix B on moodle)

For electrons in Si at 300 K (m_{ce} =0.28m_o) and v_{the} =2 × 10^7 cm/s

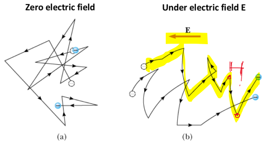
Thermal motion and scattering

But... **semiconductor crystal is not perfect**:

- Atoms themselves are vibrating around their equilibrium position in the lattice
- There are impurities and crystal imperfections.
  As carriers move around, they suffer frequent collisions: **scattering**

$L >> l_{ce}$

$L \approx l_{ce}$



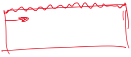
- **Mean free path**, l_{ce}: average distance travelled between collisions [cm].
- **Scattering time**, τ_{ce}: average time between collisions [s].

$$\boxed{l_{CE} = \frac{\pi}{4} v_{th} \tau_{CE}}$$

Thermal motion and scattering

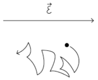
Scattering mechanisms:

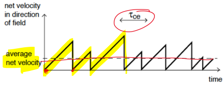
1. Lattice or **phonon scattering**: carriers collide with vibrating lattice atoms
   phonon absorption and emission
   ⇒ some energy exchanged (~ tens of meV)
2. **Ionized impurity scattering**: Coulombic interaction between charged impurities and carriers
   ⇒ no energy exchanged
3. **Neutral impurity scattering** with neutral dopants, interstitials, vacancies, etc
4. **Surface scattering** in inversion layer (surface roughness)
5. **Carrier-carrier scattering (important when carrier concentration is high)**
   Order of magnitude of τ_c < 1 ps
   Then, order of magnitude of l_c < 50 nm



Drift

In the presence of an electric field, electrons drift:





$\vec{F} = -q.\vec{E}$

$\vec{F} = m.\vec{a}$

$\frac{m.\vec{v}_e}{\tau_{ce}} = -q\vec{E}$

$\vec{v}_e = -\frac{q.\vec{E}.\tau_{ce}}{m^*}$

μ_e

Drift velocity

$$v_e^{drift} = -\frac{q\mathcal{E}\tau_{ce}}{m^*_{ce}}$$

$$v_e^{drift} = -\mu_e \mathcal{E}$$

$$\mu_e \equiv \text{electron mobility } [cm^2/V \cdot s]$$

Electron mobility: Corresponds to the **ease of carrier motion** in response to *E*.
It depends on the **strength of the scattering mechanisms**.

Drift

In the presence of an electric field, electrons drift:

$$v_e^{drift} = -\mu_e \mathcal{E}$$

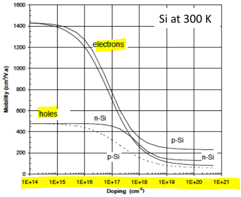
$$v_h^{drift} = \mu_h \mathcal{E}$$

$I = q . \underline{n} . v$

$= q . n . \mu . E$

Mobility depends on:

- **doping level**
- whether carrier is **majority** or **minority-type**.



at low *n*:

- Mobility is limited by phonon scattering
- thus independent of doping.

at high *n*:

- Mobility is limited by ionized impurity scattering;
- It is not a strong function of the type of dopant, but only on its concentration.
- Typically, attractive scattering is more pronounced then repulsive: majority carriers have lower mobility

Drift

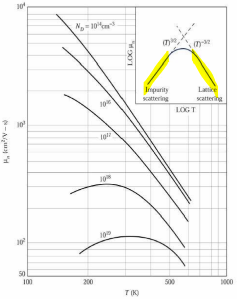
Increasing temperature and increasing doping results in reduction of mobility.

Increasing temperature: increases the number of phonons, which increases the probability that an electron will be scattered by a phonon.

Increasing doping: each dopant atom can scatter electrons.

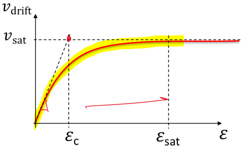
Thus:

higher doping level -> lower mobility
higher temperature -> lower mobility

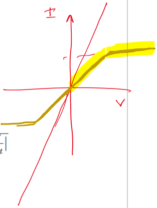


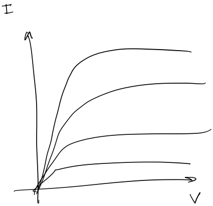
Velocity saturation

The **linear relationship** between drift velocity and electric field **is no longer valid at high fields**



$v_e = -\mu_e \cdot E$





Drift velocity vs. electric field fairly well described by:

$$v^{drift} = \mp \frac{\mu \mathcal{E}}{1 + \left| \frac{\mu \mathcal{E}}{v_{sat}} \right|}$$

Field required to saturate velocity:

$$\mathcal{E}_c = \frac{v_{sat}}{\mu}$$

$$\mathcal{E}_{\text{sat}} = 9\mathcal{E}_{\text{c}}$$

Velocity saturation crucial in modern devices:

if $\mu = 500\ cm^2/V.s$, $\mathcal{E}_{sat} = 2 \times 10^4\ V/cm$ (2V across 1 μm)
Since μ depends on doping, $\mathcal{E}_{sat}$ depends on doping too.

Velocity saturation

The **linear relationship** between drift velocity and electric field **breaks at high fields**

Implicit assumption:
quasi-equilibrium, that is, scattering rates not much affected from equilibrium.

$$v^{drift} \sim \mathcal{E} \quad \text{only if} \quad v^{drift} \ll v_{th}$$

For **high *E***: carriers acquire substantial energy from *E*
→ optical phonon emission strongly enhanced

scattering time: $\sim\ 1/\mathcal{E}$

Drift velocity saturates at

$$\underline{v_{sat}} \simeq \sqrt{\frac{8}{3\pi}\frac{E_{opt}}{m_c^*}}$$

Mobility depends on doping level and whether carrier is majority or minority-type.

For Si at 300 *K*:

- $v_{sat} \sim 10^7\ cm/s$ for electrons
- $v_{sat} \sim 6 \times 10^6\ cm/s$ for holes

Drift current

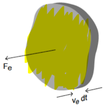
Particle flux and current density

particle flux ≡ # particles crossing unity surface (normal to flow) per unit time [$cm^{-2} \cdot s^{-1}$]

current density ≡ electrical charge crossing unity surface (normal to flow) per unit time [$C \cdot cm^{-2} \cdot s^{-1}$]

$$J_e = -qF_e$$

particle flux: $$F_e = \frac{nv_e dt}{dt} = nv_e$$

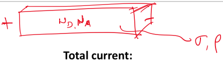


Electron and hole current density:

$$[C/s\,cm^2]\ J_e = \underbrace{-qn}_{C/cm^3}\underbrace{v_e}_{cm/s} = \underbrace{\left(\frac{C}{s}\right)}_{A} \cdot \frac{1}{cm^2}$$

$$J_h = qpv_h$$

Drift current



σ, ρ

Drift current (low fields):

$$J_e = q\mu_e n\mathcal{E}$$

$$J_h = q\mu_h p\mathcal{E}$$

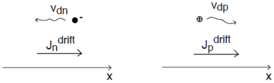
Total current:

$$J = \overbrace{q(\mu_e n + \mu_h p)}\mathcal{E}$$

$[I/cm^2]$ Ohm's law for semiconductors $[V/cm]$

Electrical conductivity [$(\Omega \cdot cm)^{-1}$]: $$\sigma = q(\mu_e n + \mu_h p)$$

Electrical resistivity [$\Omega \cdot cm$]: $$\rho = \frac{1}{q(\mu_e n + \mu_h p)} = \frac{1}{\sigma}$$



Drift current

ρ strong function of doping ⇒

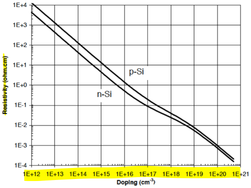
frequently used by wafer vendors to specify doping level of substrates

$$\text{-for n-type: } \rho_n \simeq \frac{1}{q\mu_e N_D}$$

$$\text{-for p-type: } \rho_p \simeq \frac{1}{q\mu_h N_A}$$

Si at 300K:

ρ



Drift current

Drift current (**high fields**):

$$J_{esat} = qnv_{esat}$$

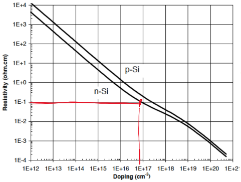
$$J_{hsat} = qpv_{hsat}$$

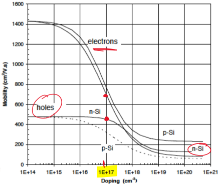
The only way to get **more current at high fields** is to **increase carrier concentration**.

Exercice

Estimate the electrons and holes drift velocity, for a n-type Si with resistivity of 0.1 ohm cm and under an applied field of 1000 V/cm.

Si at 300K:





$N_D = 9 \times 10^{16}\ cm^{-3}$

$$\sigma = \frac{1}{q \cdot n \cdot \mu_e}$$

$1k\Omega$

$n_0 = N_D = 9 \times 10^{16}\ cm^{-3}$

$P_0 = \frac{n_i^2}{n_0} = \frac{10^{20}}{9 \times 10^{16}} = 1.1 \times 10^{3}\ cm^{-3}$

$\mu_e = 750\ cm^2/Vs$

$\mu_h = 460\ cm^2/Vs$

$v_e^{drift} = -\mu_e \cdot E = -750\ cm^2/Vs \cdot 1000\ V/cm = -7.5 \times 10^5\ cm/s$

$v_h^{drift} = +\mu_h \cdot E = +460\ cm^2/Vs \cdot 1000\ V/cm = 4.6 \times 10^5\ cm/s$

$J_e^{drift} = -q \cdot n \cdot v_e^{drift} = -1.6 \times 10^{-19}\ C \times 9 \times 10^{16} \times (-7.5 \times 10^5) = 1.1 \times 10^4\ A/cm^2$

$J_h^{drift} = +q \cdot P \cdot v_h^{drift} = +1.6 \times 10^{-19}\ C \cdot 1.1 \times 10^3 \times 4.6 \times 10^5 = 9 \times 10^{-11}\ A/cm^2$

$J^{Drift} = J_e^{Drift} + J_h^{Drift} \simeq J_e^{Drift}$

Drift current

Sheet resistance

The sheet resistance concept is used to characterize both wafers and thin doped layers, since it is typically **easier to measure than the resistivity** of the material.

The sheet resistance of a uniformly doped layer with resistivity, ρ, and thickness, t, is given by their ratio:

$$Rs = \frac{\rho}{t}$$

While the unit of the sheet resistance is Ohms, one refers to it as **Ohms per square**.

This nomenclature comes in handy when the resistance of a rectangular piece of material with **length, L**, and **width W** must be obtained. It equals the product of the sheet resistance and the number of squares or:

$$R = Rs\frac{L}{W}$$

Transmission line method (TLM)

technique to determine the sheet resistance and contact resistance

The sheet resistance of a doped layer with resistivity, ρ, and thickness, t, is:

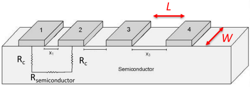
$$Rs = \frac{\rho}{t}$$

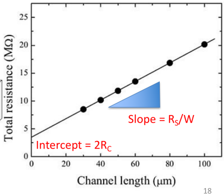
The resistance of a rectangular piece of material with length, L, and width W

$$R = Rs\frac{L}{W}$$

Taking into account the contact resistance (R_c):

$$R = 2R_c + R_s\frac{L}{W}$$

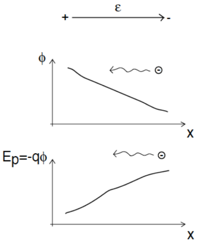




Energy band diagram under electric field

Energy band diagram under electric field:

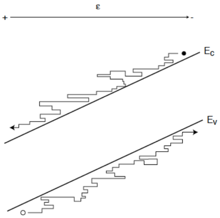
Energy band diagram needs to account for **potential energy of electric field**



Electron trades potential energy by kinetic energy as it moves to the left
→ *total electron energy unchanged*

Energy band diagram under electric field

Energy band diagram is **picture of electron energy**
⇒ must add E_p to semiconductor energy band diagram ⇒ bands tilt: **Band bending**



Measuring from an arbitrary energy reference:

$$E_c + E_{ref} = E_p = -q\phi$$

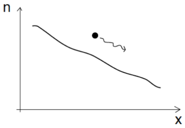
$$\mathcal{E} = -\frac{d\phi}{dx} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx}$$

Shape of energy bands = shape of φ with a minus sign.
Can easily compute E from energy band diagram.

Diffusion

Movement of particles from regions of **high concentration to** regions of **low concentration**.

Diffusion produced by collisions with background medium (*i.e.*, vibrating Si lattice).



Diffusion flux is proportional to the gradient of concentration [Fick's first law]

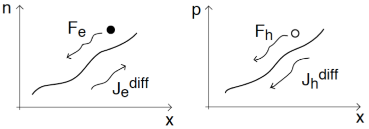
$$F_e = -D_e \frac{dn}{dx}$$
$$F_h = -D_h \frac{dp}{dx}$$

$$D \equiv \text{diffusion coefficient } [cm^2/s]$$

Diffusion

Diffusion current: Electrons and holes are charged particles and their diffusion creates current

$$F_e = -D_e \frac{dn}{dx} \qquad J_e = qD_e \frac{dn}{dx}$$
$$F_h = -D_h \frac{dp}{dx} \qquad J_h = -qD_h \frac{dp}{dx}$$



Non-uniformly doped semiconductor in thermal equilibrium

In summary:

μ relates to ”ease” of carrier drift in an electric field.

D relates to ”ease” of carrier diffusion as a result of a concentration gradient.

Is there a relationship between μ and D?

Non-uniformly doped semiconductor in thermal equilibrium

In summary:

μ relates to ”ease” of carrier drift in an electric field.

D relates to ”ease” of carrier diffusion as a result of a concentration gradient.

Is there a relationship between μ and D?

Yes, Einstein relation:

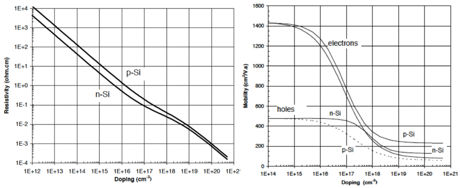
$$\frac{D_e}{\mu_e} = \frac{D_h}{\mu_h} = \frac{kT}{q}$$

Relationship between D and μ only depends on T.

Exercice

Estimate the diffusion coefficients for electrons and holes, for a p-type Si with resistivity of 0.1 ohm cm

Si at 300K:



Key conclusions

At finite temperatures, carriers move around in a **random way** suffering many collisions: ***thermal motion***.

Dominant scattering mechanisms in bulk Si at 300K: **phonon scattering** and **ionized impurity scattering**.

Two processes for carrier flow in semiconductors: **drift and diffusion**.

General relationship between carrier net velocity (by drift or diffusion) and current density:

$$J_e = -qnv_e \qquad J_h = qpv_h$$

For low fields, $v_{drift} \sim E$
For high fields, $v_{drift} \sim v_{sat}$

Driving force for **diffusion**: **concentration gradient**.

Order of magnitude of key parameters for Si at 300K:

– electron mobility: $\mu_e \sim 100 - 1400\ cm^2/V \cdot s$
– hole mobility: $\mu_h \sim 50 - 500\ cm^2/V \cdot s$
– saturation velocity: $v_{sat} \sim 10^7\ cm/s$

– $v_{th} \sim 2 \times 10^7\ cm/s$
– $\tau_c < 1\ ps$
– $l_c < 50\ nm$